



# REFLECTIONS ON AIRCRAFT UNMASK RANGES

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6 October 1981



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An analytic representation is obtained of the rate following aircraft unmasks to an air defense site expressed as a distribution that depends on a singunmask range. That parameter is given as a functistics, aircraft height, and air defense site located	. The unmask range is gle parameter, the mean ion of terrain character-			

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## REFLECTIONS ON AIRCRAFT UNMASK RANGES

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# 1. INTRODUCTION.

- a. This paper was delivered at the Army Operations Research Sumposium XX on 6 October 1981. The paper is a sanitized version of a classified paper which was published on 26 May 1981 and which U.S. Government personnel can obtain from the Defense Technical Information Center through accession number ADC025223L.
- b. The purpose of this paper is to develop an analytic representation of the range at which a terrain-following aircraft first becomes unmasked to an air defense (AD) site. The paper is an update to unpublished work performed by the second author, D. C. Hardison, in 1977 in support of the Division Air Defense Gun Cost and Operational Effectiveness Analysis (DIVAD Gun COEA). The methodology developed for the DIVAD Gun COEA is included in the ADAGE Simulation. That simulation was developed by the U.S. Army Materiel System Analysis Activity and is used currently by the U.S. Army Air Defense School for AD studies.
- c. Basic to the representation to be developed here are the following precepts:
- (1) Because unmask ranges vary, a distribution of ranges must be sought, and appropriate statistics must be obtained.
- (2) The distribution of unmask ranges depends on geographic locations due to the effects of terrain shielding.
- (3) The distribution of unmask ranges depends on aircraft height above the ground.
- (4) The distribution of unmask ranges depends on AD weapon site selection. Favored sites are assigned to some systems because they are few in number or have high priority, while less ideal sites are given to other systems because they are more numerous or are placed for reasons other than maximization of field-of-fire.

#### 2. CONSTANT MASK ANGLE APPROACH.

a. A simple representation of unmask ranges is provided by the constant mask angle approach. In Figure 1, aircraft height H is measured above mean terrain altitude;  $\alpha$  is the mask angle; and the AD weapon is placed at mean terrain altitude. The unmask range R is given by

$$R = \frac{B}{\sin \alpha} . \tag{1}$$

This equation yields the values shown in Table 1. Note that it was common practice in the early 1950s to assume an average mask angle of 30 mils. Note also that the simple mask angle model of equation (1) is obviously not satisfactory for an aircraft close to, or on, the surface; indeed, according to the model, an aircraft at H=0 would unmask at R=0.

Figure 1. Constant Mask Angle, Aircraft Height Measured Above Mean Terrain.

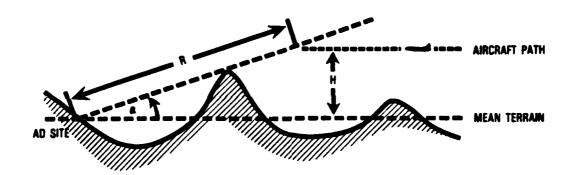


Table 1. Unmask Range (Kilometers)--Constant Mask Angle, Aircraft
Height Measured Above Mean Terrain.

Mask Angle	Aircraft Height (Meters)						
(Mils)	100	200	300	400			
15	6.67	13.33	20.00	26.67			
30	3.33	6.67	10.00	13.33			
60	1.67	3.33	5.00	6.67			

b. If aircraft height is measured above peak-to-peak (a gross approximation to terrain-following), then the constant mask angle approach can be used to obtain an expression for unmask range provided that the distance  $R_p$  to the first obscuring peak is known. From Figure 2, the unmask range R is

given by

$$R = R_p + \frac{H}{\sin \alpha} . \tag{2}$$

Treating  $\mathbf{R}_{\mathbf{P}}$  parametrically and applying equation (2) yield the values shown in Table 2. Note that with the constant mask angle approach—illustrated in Tables 1 and 2—the unmask range R varies linearly with aircraft height H.

Figure 2. Constant Mask Angle, Aircraft Height Measured Above Peak-to-Peak.

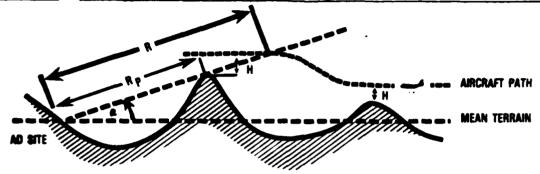


Table 2. Unmask Range (Kilometers)--Constant Mask Angle, Aircraft Height
Measured Above Peak-to-Peak.

R <sub>p</sub> (Kilometers)	Mask Angle (Mils)	100	Aircra 200	ft Height 300	(Meters) 400
1.0	15	7.67	14.33	21.00	27.67
	30	4.33	7.67	11.00	14.33
	60	2.67	4.33	6.00	7.67
2.0	15	8.67	15.33	22.00	28.67
	30	5.33	8.67	12.00	15.33
	60	3.67	5.33	7.00	8.67
3.0	15	9.67	16.33	23.00	29.67
	30	6.33	9.67	13.00	16.33
	60	4.67	6.33	8.00	9.67

### 3. PETERSON WORK.

a. In connection with studies of the range of engagements of tank combat in World War II, R. H. Peterson suggested in 1951 that surface-to-surface line-of-sight distances can be represented by a simple frequency distribution that is fully specified by a single parameter, the mean value  $\overline{R}$ . Peterson suggested the function

$$f(R) = \left(\frac{4R}{R^2}\right) \exp\left(-\frac{2R}{R}\right) \tag{3}$$

as the distribution of distances between obscuring objects. The corresponding cumulative distribution function F(R) is given by

$$\mathbf{F}(\mathbf{R}) = \int_0^{\mathbf{R}} \mathbf{f}(\mathbf{r}) d\mathbf{r} = 1 - \mathbf{G}(\mathbf{R}) \tag{4}$$

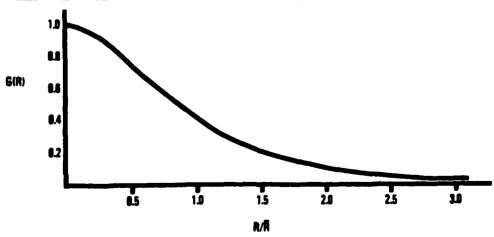
where

$$G(R) = \left(1 + \frac{2R}{\overline{R}}\right) \exp\left(-\frac{2R}{\overline{R}}\right). \tag{5}$$

The function G(R) represents the probability of line-of-sight from a given location to at least the range R. A graph of G(R) is shown in Figure 3. The contiguous region visible from a given site can be projected onto a horizontal plane to yield an area

$$\overline{A} = \int_0^\infty G(R) \cdot 2 \pi R dR = \frac{3}{2} \pi \overline{R}^2 . \qquad (6)$$

Figure 3. Unmask Distribution Function G(R).



- b. In 1953, D.C. Hardison, R.H. Peterson, and A.H. Benvenuto analyzed topographic maps for Northwest Europe to determine the distances from ground weapon sites to obstacles (e.g., woods, hills, built-up areas) capable of providing concealment to moving ground vehicles. The emphasis was on finding the "sighting range" from a weapon site to the nearest obstacle that could provide concealment. The following conclusions were drawn:
- (1) For a single geographic area, equation (3) provides a good approximation to the distribution of sighting ranges.
- (2) Sighting ranges vary greatly from one area to another. This is a natural consequence of the variability in terrain.
- (3) If  $\overline{R}$  denotes the mean sighting range for an area, the distribution of  $\overline{R}$  across the various areas of Northwest Europe can be approximated by a Gaussian distribution with mean  $\mu_{\overline{R}}$  and standard deviation  $\sigma_{\overline{R}}$  satisfying

$$\mu_{\overline{R}} = 0.68 \text{ kilometers}$$
 (7)

$$\sigma_{\overline{R}} \approx 0.20 \; \mu_{\overline{R}} \tag{8}$$

- c. In order to apply the Peterson representation to the unmask ranges of terrain-following aircraft, the following assumptions are made: The mean unmask range  $\overline{R}$  of a terrain-following aircraft depends on terrain characteristics, aircraft height above the ground, and AD site location. Given  $\overline{R}$ , the function G(R) of equation (5) can be used to represent the probability of unmask at a range at least R from the AD site. With these assumptions, the remaining task is to obtain an analytic expression for  $\overline{R}$ .
- 4. CAYWOOD-SCHILLER WORK.
- a. In work performed by analysts of Caywood-Schiller Associates in 1964 and 1965, detailed examinations were made of unmask ranges of low-altitude aircraft for AD sites in several areas in Germany and Korea. Consistent with the earlier work of Hardison, Peterson, and Benvenuto, the ranges were found to differ widely from area to area, and, as expected, the constant mask angle model was found to be unsatisfactory for aircraft flying at very low altitudes.
- b. The following equation, drawn from the work of Caywood-Schiller Associates, expresses—in terms of terrain characteristics, aircraft height above the

ground, and AD site location—the mean continguous area  $\overline{A}$  of the projection onto the horizontal plane of the surface that would be visible to the AD site:

$$\frac{\mathbf{Z} = \frac{287 \exp(0.42((H/\sigma) + 1.3X))}{(\beta\sigma)^{0.85}} \tag{9}$$

where

 $\overline{A}$  = mean contiguous area visible from the AD site (km<sup>2</sup>),

H = aircraft height above the ground (meters),

 $\sigma$  = standard deviation of terrain altitude (meters),

X = AD site height above mean terrain altitude, expressed in multiples of  $\sigma$ , and

 $\beta$  = reciprocal of mean peak-to-peak distance (km<sup>-1</sup>).

### 5. UNMASK RANGE MODEL.

a. The work of Peterson and that of Caywood-Schiller Associates can be combined to yield a representation of unmask ranges for terrain-following aircraft. Under the assumption that the Peterson representation applies to the case of aircraft unmask, equations (6) and (9) yield the following expression for the mean unmask range  $\overline{R}$  (kilometers) of a terrain-following aircraft:

$$\bar{R} = \frac{7.804 \exp(0.21((H/\sigma) + 1.3X))}{(\beta\sigma)^{0.425}}.$$
 (10)

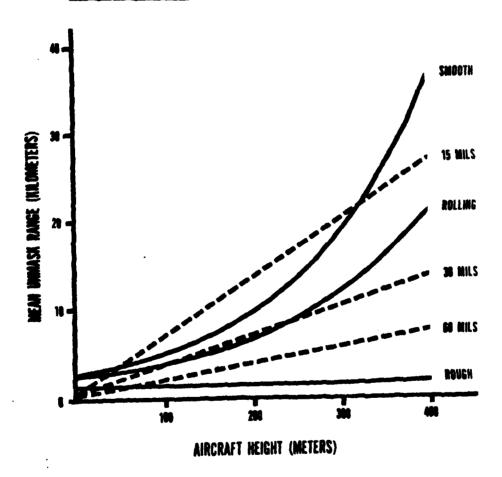
Equations (10) and (5) provide the sought representation; i.e.,  $\beta$ ,  $\sigma$ , H, and X determine  $\overline{R}$  through equation (10);  $\overline{R}$  determines the distribution function G(R) through equation (5); and G(R) provides the probability of unmask at a range at least R.

b. In order to apply the representation, typical values of  $\beta$  and  $\sigma$ , drawn from the work of Caywood-Schiller Associates, are shown in Table 3. Also shown are characterizations of terrain as "smooth," "rolling," or "rough", and mean unmask ranges from equation (10) for an AD site at mean terrain altitude (i.e., X=0). The data from Table 3 are plotted as solid curves in Figure 4; note the non-linear behavior. For comparison purposes, the data from Table 2 (with  $R_D=1.0$  kilometer) are plotted as dashed lines.

Table 3. Mean Unmask Range (Kilometers)--Typical Terrain Types, AD Site at Mean Terrain Altitude.

Terrain Type	ß	σ	A: 0	ircraft 100	Height 200	(Meter 300	s) 400
smooth	0.40	32	2.64	5.09	9.81	18.91	36.46
rolling	0.52	37	2.22	3.92	6.91	12.19	21.50
rough	0.59	165	1.11	1.27	1.44	1.63	1.86

Figure 4. Mean Unmask Range--Typical Terrain Types, AD Site at Mean Terrain Altitude.



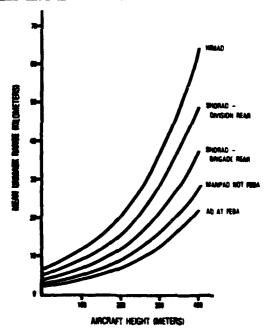
# 6. ILLUSTRATIONS.

a. In order to illustrate the impact of weapon site location, mean unmask ranges  $\overline{R}$  are shown in Table 4 for various values of the parameter X= site height above mean terrain altitude expressed in multiples of the standard deviation of terrain altitude. For this example, the terrain parameters for the rolling terrain in Table 3 are used. The data from Table 4 are plotted in Figure 5.

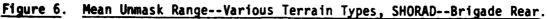
Table 4. Mean Unmask Range (Kilometers)--Various Site Locations, Rolling Terrain.

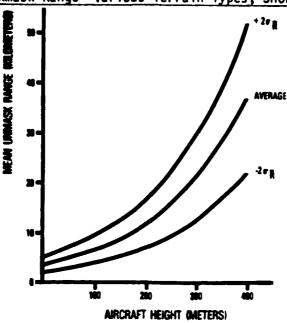
AD System	Site Height X	Ai O	rcraft 100	_	(Meters 300	) 400
Organic, AD at FEBA AD w/convoys	0	2.22	3.92	6.91	12.19	21.50
MANPAD not FEBA, AD in reserve	1	2.92	5.15	9.08	16.02	28.25
SHORADbrigade rear	2	3.83	6.76	11.93	21.04	37.12
SHORADdivision rear & corps	3	5.04	8.89	15.67	27.65	48.77
HIMAD	4	6.62	11.68	20.60	36.33	64.08

Figure 5. Mean Unmask Range--Various Site Locations, Rolling Terrain.



- The relationship of the values in Table 4 to specific AD weapon systems requires discussion. Intuition, and work by R.R. Hare, G.G. Wahba, and V.N. Behrns, lead to the conclusion that some AD units can be sited more favorably than others. For example, organic weapons (e.g., tanks, infantry carriers, antitank missiles) would likely be on the roads or other trafficable terrain. Similarly, AD weapons protecting maneuver units at the forward edge of the battle area (FEBA), and those moving with convoys, would likely be at, or perhaps even below, the local mean terrain altitude. Assuming X = 0 for such organic and AD weapons seems reasonable. Man-portable air defense (MANPAD) systems and other AD weapons providing local defense of units not directly engaged should be able to achieve the local high ground; say, X = 1. Short-range air defense (SHORAD) missile systems protecting brigade rear targets should be able to attain still better sites; say, X = 2. Similarly, SHORAD systems protecting division rear and corps area assets should be located with X = 3. Finally, high-to-medium air defense (HIMAD) systems, being few in number, should get the best coverage; perhaps X = 4is appropriate for these systems.
- c. To explore the extent to which local cases might differ from the average, note in Table 3 that the rolling terrain provides unmask ranges midway between those of the smooth and rough terrains. Assume, therefore, that the unmask ranges  $\overline{R}$  of equation (10) for that rolling terrain represent average values  $\mu_{\overline{R}}$  for a theater. Assume also that for fixed values of X and H, the distribution of aircraft mean unmask ranges  $\overline{R}$  across terrain types is approximately Gaussian and satisfies equation (8). Choosing SHORAD defending brigade rear assets as an example, and using  $\pm 2~\sigma_{\overline{R}}$  as illustrative, yield the curves plotted in Figure 6. The "average" curve is carried forward from Figure 5. Note that the  $\mu_{\overline{R}}$  2  $\sigma_{\overline{R}}$  case corresponds to  $\overline{R}\approx 0.60~\mu_{\overline{R}}$ , while the  $\mu_{\overline{R}}$  +2  $\sigma_{\overline{R}}$  case corresponds to  $\overline{R}\approx 1.40~\mu_{\overline{R}}$ , where  $\mu_{\overline{R}}$  is the average value.





### 7. SUMMARY.

- a. Terrain characteristics, aircraft height above the ground, and AD site location determine the mean unmask range  $\overline{R}$  of a terrain-following aircraft through the Caywood-Schiller Associates' representation in equation (10). The parameter  $\overline{R}$  then determines the distribution G(R) of unmask ranges through the Peterson representation in equation (5).
- b. This approach to unmask ranges can be applied in a number of ways, depending on the knowledge of terrain characteristics; for example:
- (1) If values of  $\beta$  and  $\sigma$  are known or can be estimated, then equation (10) can be applied directly.
- (2) If only a general characterization of the terrain as smooth, rolling, or rough is known, then appropriate values of  $\beta$  and  $\sigma$  from Table 3 can be used in equation (10).
- (3) If various types of terrain for a theater must be treated, then a Gaussian distribution of mean unmask ranges can be assumed, with the average obtained from the rolling terrain of Table 3 and the standard deviation obtained from equation (8), as was done for Figure 6.

